LA-UR-21-22445
Approved for public release; distribution is unlimited.

Title: Qubit-efficient entanglement spectroscopy using qubit resets

Author(s): Subasi, Yigit
Jirka, Justin

Intended for: APS March Meeting, 2021-03-15/2021-03-19 (Virtual, Maryland, United States)

Issued: 2021-03-25 (rev.1)

Disclaimer:
Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by Triad National Security, LLC for the National Nuclear Security Administration of U.S. Department of Energy under contract 89233218CNA000001. By approving this article, the publisher recognizes that the U.S. Government retains nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

Qubit-efficient entanglement spectroscopy using qubit resets

Yiğit Subaşı

Los Alamos National Laboratory

Joint work with: Justin Yirka

The University of Texas at Austin

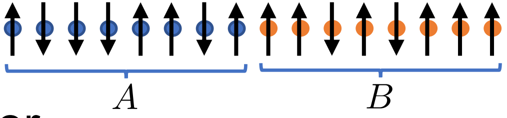
APS MARCH MEETING 3/16/21

LA-UR-21-22445

Entanglement spectroscopy

A pure bipartite quantum state is entangled if it ***can not*** be written as

$$|\psi\rangle_{AB} \neq |\psi_A\rangle \otimes |\psi_B\rangle$$



The reduced state is mixed and given by a density operator

$$\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|_{AB})$$

The largest eigenvalues $\lambda_1 \geq \lambda_2 \cdots \geq \lambda_r$ of ρ_A contain information about the nature of entanglement between subsystems A and B. [Li, Haldane 08]

Newton-Girard Formula

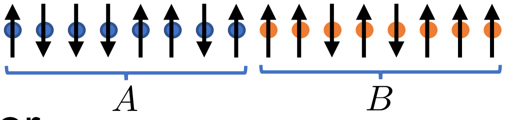
[Song *et al.* 2012]

$$\{\mathrm{Tr}(\rho_A^n)\}_{n=1,\dots,r} \Longrightarrow \lambda_1 \geq \lambda_2 \cdots \geq \lambda_r$$

Entanglement spectroscopy

A pure bipartite quantum state is entangled if it ***can not*** be written as

$$|\psi\rangle_{AB} \neq |\psi_A\rangle \otimes |\psi_B\rangle$$



The reduced state is mixed and given by a density operator

$$\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|_{AB})$$

The largest eigenvalues $\lambda_1 \geq \lambda_2 \cdots \geq \lambda_r$ of ρ_A contain information about the nature of entanglement between subsystems A and B. [Li, Haldane 08]

(Quantum Computer)

(Classical Computer)

Newton-Girard Formula

[Song *et al.* 2012]

$$\{\mathrm{Tr}(\rho_A^n)\}_{n=1,\ldots,r} \longrightarrow \lambda_1 \geq \lambda_2 \cdots \geq \lambda_r$$

Key observation: $\mathrm{Tr}(\rho_A^n) = \langle\psi|^{\otimes n}\, \pi_A^{\text{cyclic}}\, |\psi\rangle^{\otimes n}$

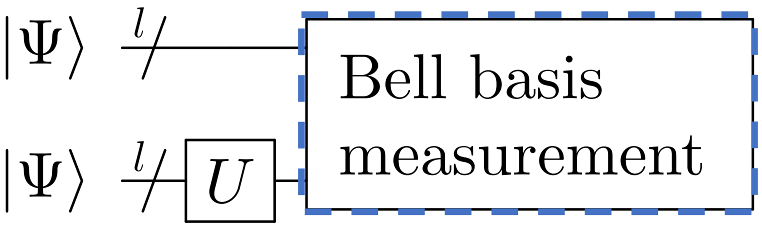
Previous algorithms for $\mathrm{Tr}(\rho_A^n)$

- **Algorithm based on the Hadamard Test [Johri, Steiger, Troyer 17]**
- **Algorithm based on the Two-Copy Test [YS, Cincio, Coles 19]**
- **Other (arguably) less NISQ-friendly algorithms [Lloyd, Mohseni, Rebentrost 14], [Subramanian, M.-H. Hsieh 19], …**

Previous algorithms for $\mathrm{Tr}(\rho_A^n)$

- **Algorithm based on the Hadamard Test [Johri, Steiger, Troyer 17]**
- **Algorithm based on the Two-Copy Test [YS, Cincio, Coles 19]**
- **Other (arguably) less NISQ-friendly algorithms [Lloyd, Mohseni, Rebentrost 14], [Subramanian, M.-H. Hsieh 19], …**

Two-Copy Test



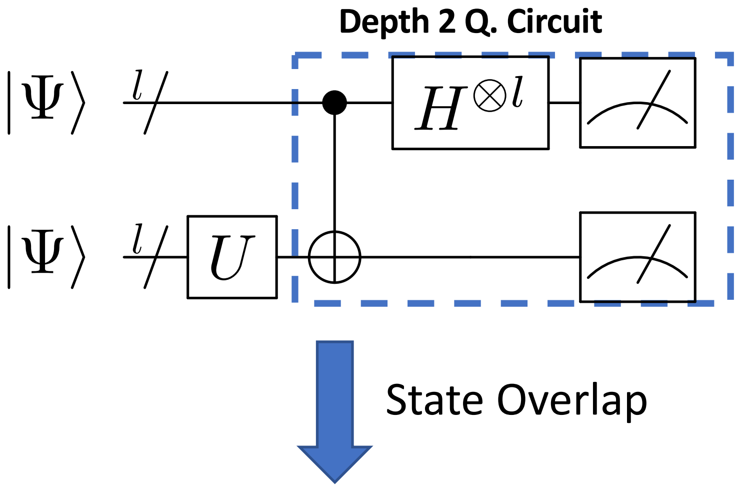
State Overlap

$$|\langle\Psi|\,U\,|\Psi\rangle|^2$$

Two-Copy Test

- Two copies of the state are needed
- All qubits are measured
- Postprocessing scales linearly in system size

YS, L. Cincio, P. Coles (2019)



$$|\langle\Psi| U |\Psi\rangle|^2$$

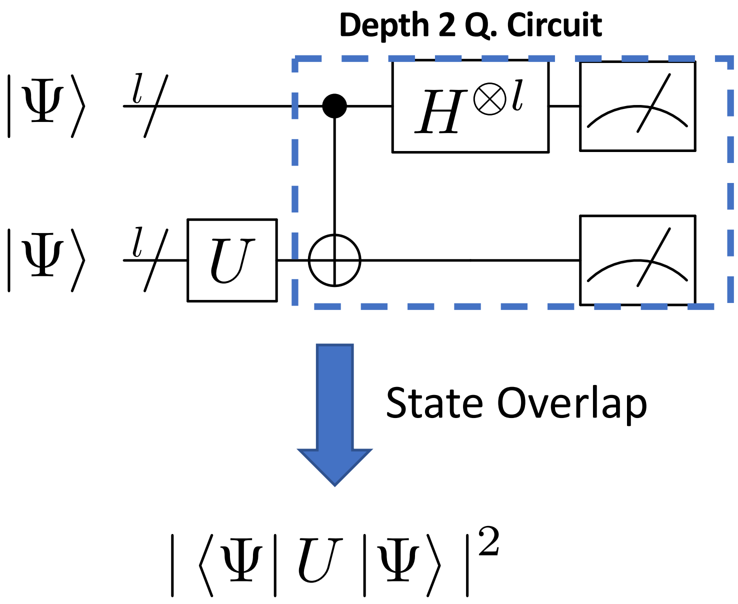
Algorithm based on two-copy test

$$\boxed{\mathrm{Tr}(\rho_A^n) = \langle\psi|^{\otimes n}\,\pi_A^{\text{cyclic}}\,|\psi\rangle^{\otimes n}}$$

We compute: $|\mathrm{Tr}(\rho_A^n)|^2 = |\,\langle\psi|^{\otimes n}\,\pi_A^{\text{cyclic}}\,|\psi\rangle^{\otimes n}\,|^2$

$$|\Psi\rangle \Longrightarrow |\psi\rangle^{\otimes n}$$

$$U \Longrightarrow \text{cyclic permutation } \pi_A^{\text{cyclic}}$$



Algorithm based on two-copy test

$$\boxed{\mathrm{Tr}(\rho_A^n) = \langle\psi|^{\otimes n}\, \pi_A^{\text{cyclic}}\, |\psi\rangle^{\otimes n}}$$

We compute: $|\mathrm{Tr}(\rho_A^n)|^2 = |\, \langle\psi|^{\otimes n}\, \pi_A^{\text{cyclic}}\, |\psi\rangle^{\otimes n}\, |^2$

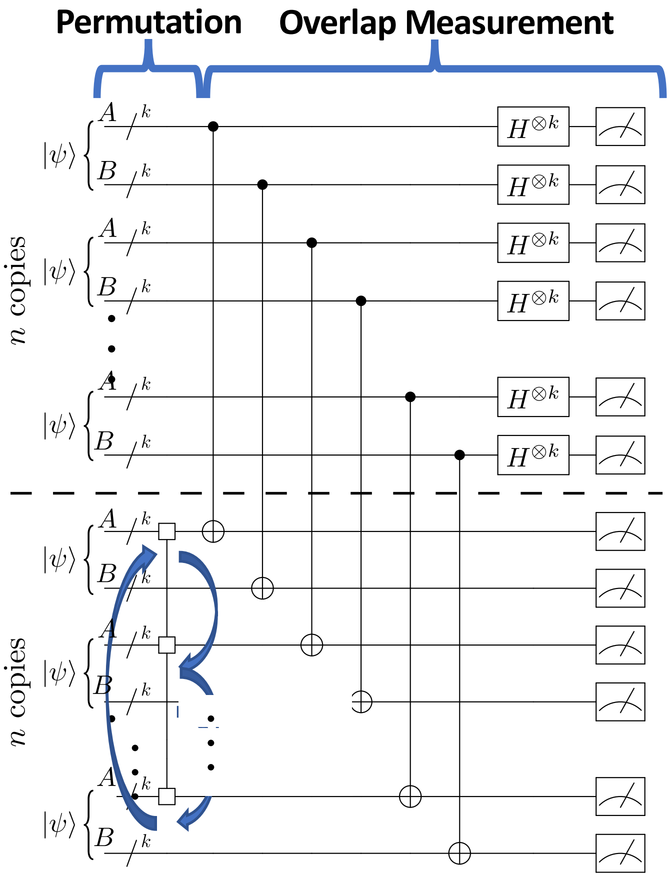
$$|\Psi\rangle \Longrightarrow |\psi\rangle^{\otimes n}$$

$$U \Longrightarrow \text{cyclic permutation } \pi_A^{\text{cyclic}}$$

This algorithm is:

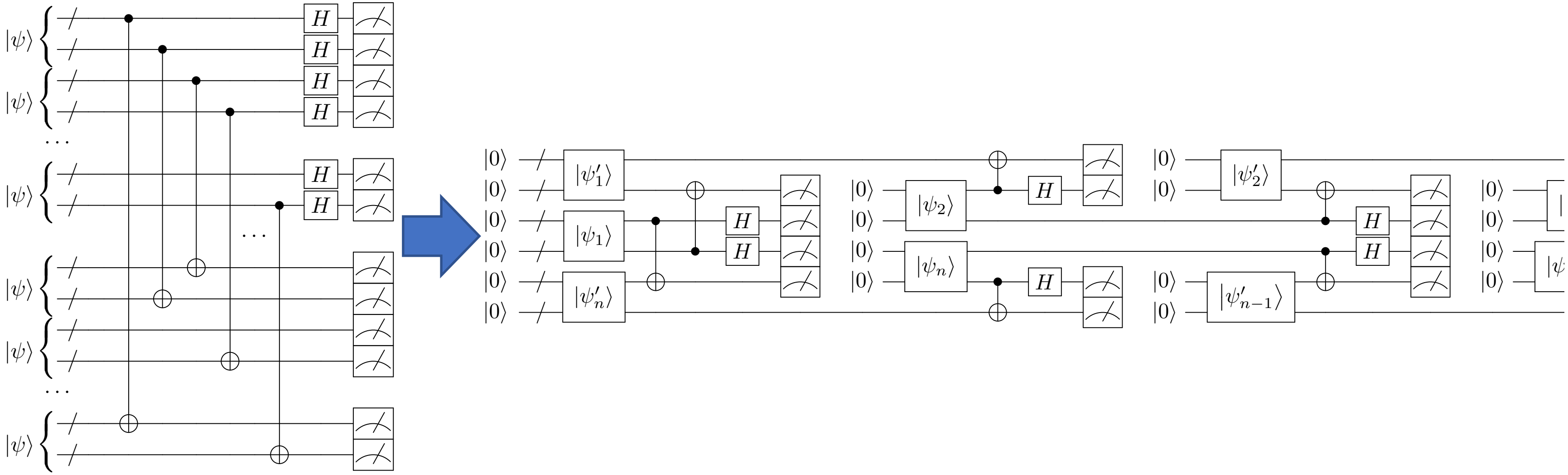
- Requires $4kn$ qubits
- Depth is 2 independent of problem size
- Postprocessing scales as $n * k$

YS, L. Cincio, P. Coles (2019)



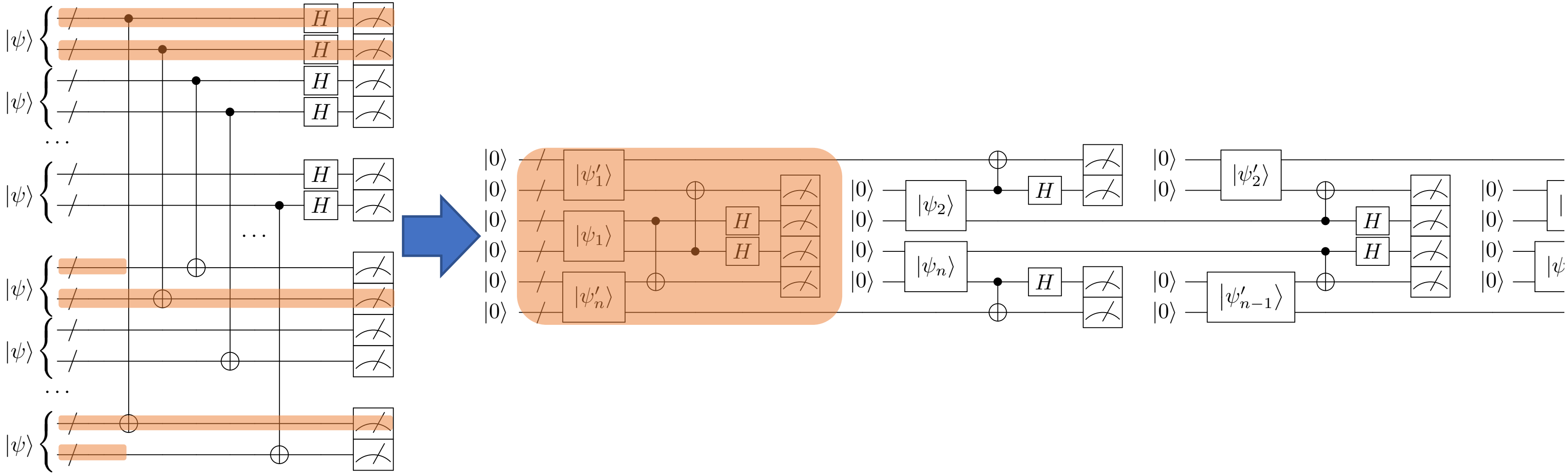
Qubit-efficient Algorithm [arXiv:2010.03080]

- **Key intuition**: When a register finishes its interactions, measure, reset, and reuse to load a new copy. Repeat as necessary for n.



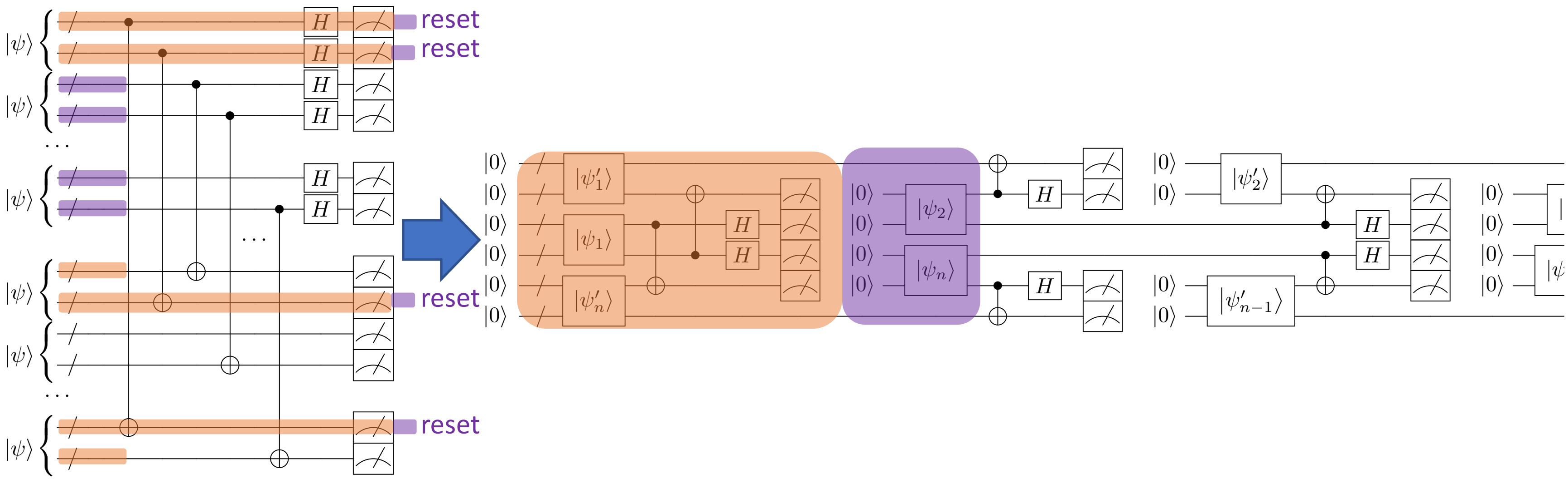
Qubit-efficient Algorithm [arXiv:2010.03080]

- **Key intuition**: When a register finishes its interactions, measure, reset, and reuse to load a new copy. Repeat as necessary for n.



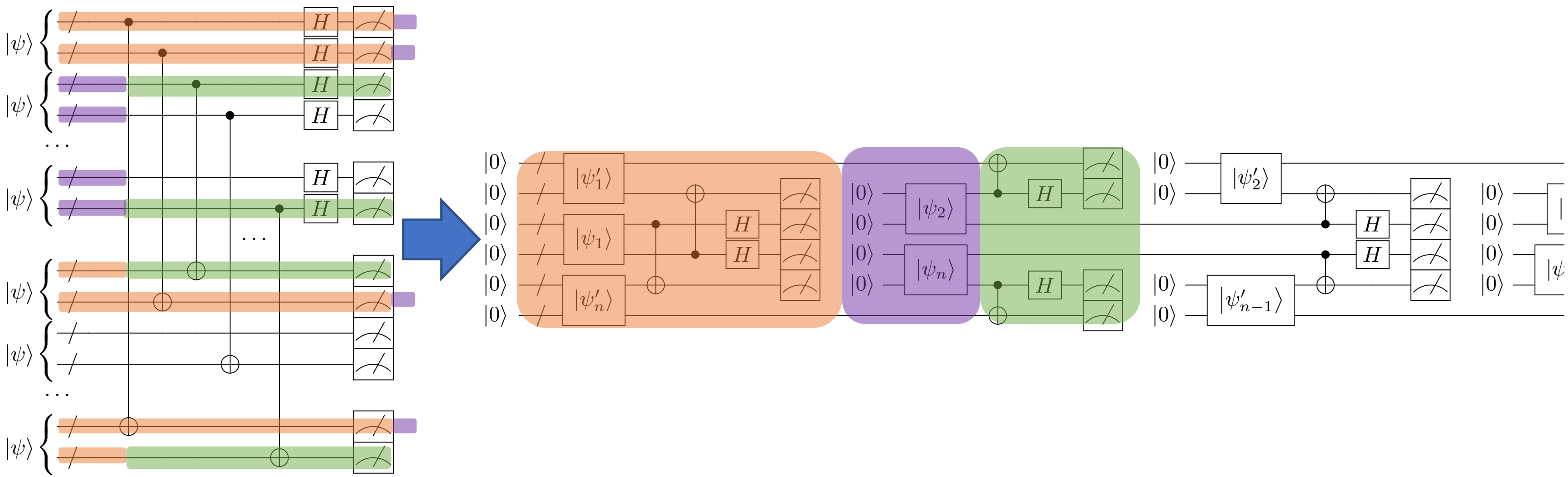
Qubit-efficient Algorithm [arXiv:2010.03080]

- **Key intuition**: When a register finishes its interactions, measure, reset, and reuse to load a new copy. Repeat as necessary for n.



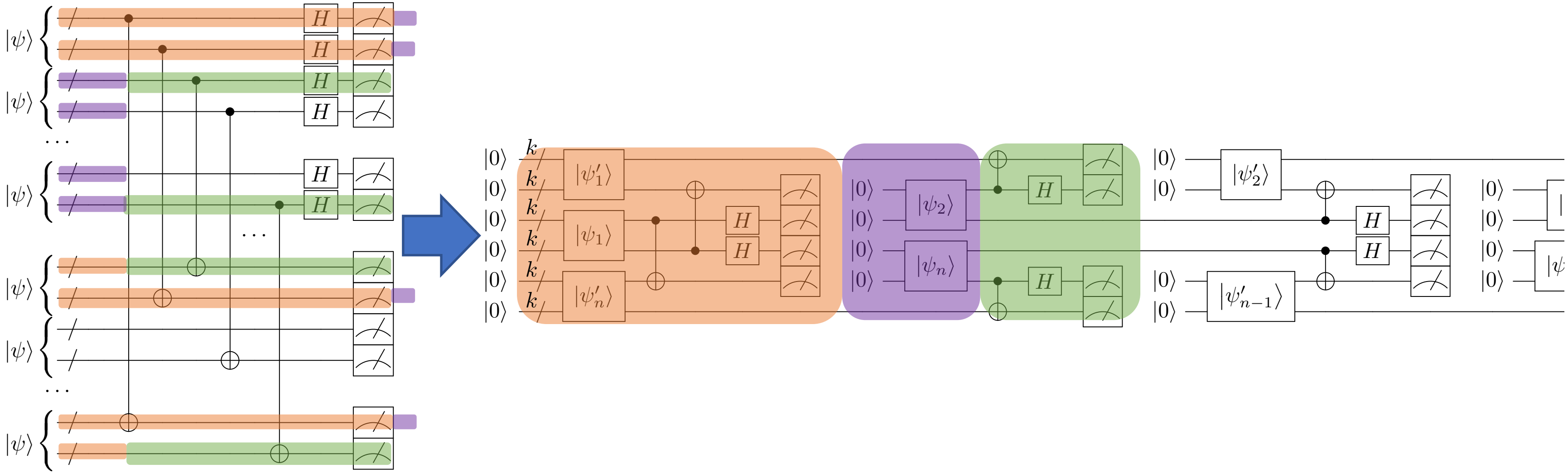
Qubit-efficient Algorithm [arXiv:2010.03080]

- **Key intuition**: When a register finishes its interactions, measure, reset, and reuse to load a new copy. Repeat as necessary for n.



Qubit-efficient Algorithm [arXiv:2010.03080]

- **Key intuition**: When a register finishes its interactions, measure, reset, and reuse to load a new copy. Repeat as necessary for n.

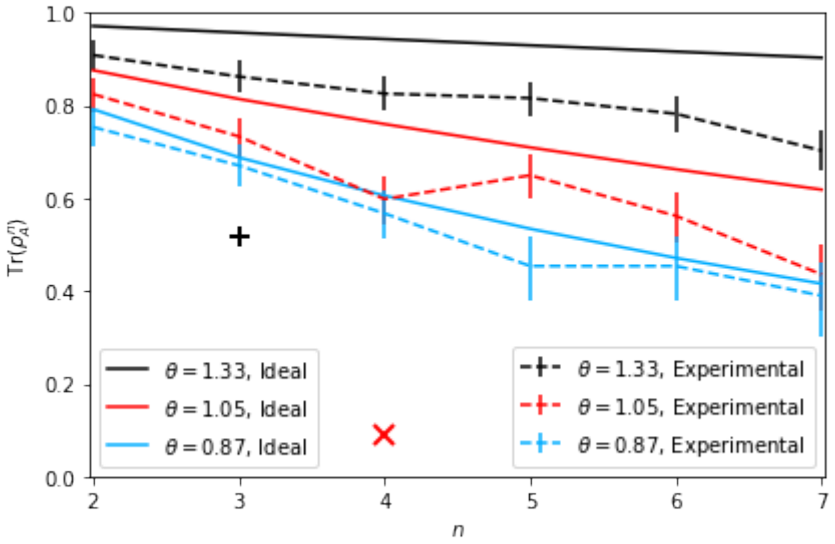


$4nk$ qubits vs $6k$ (can be improved to $4k$)

Honeywell System HO

- Tested on Honeywell 6-qubit ion trap quantum computer.
- We ran qubit-efficient algorithm on three 2-qubit states for $n = 2, \ldots, 7$.

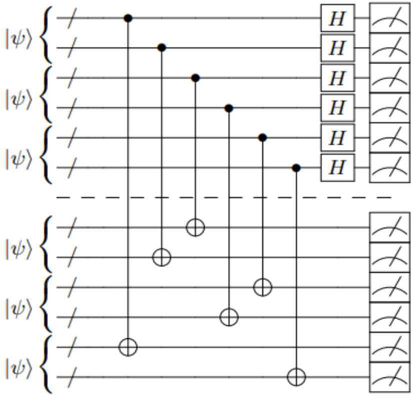
The original algorithm would require 28 qubits, more than the 6 available.



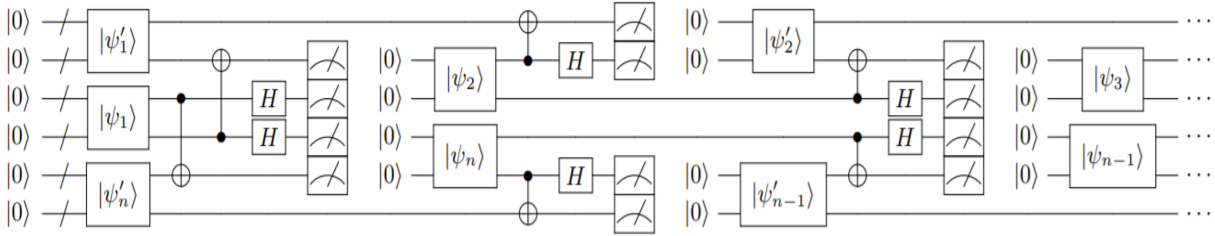
Effective Depth

- Numerical analysis shows qubit-efficient algorithms perform similar under noise.
- Circuit depth is a good heuristic in general. Deeper circuits perform worse.
- Depth can be a bad heuristic for circuits using resets.
  - Evidence: Our circuits!
  - Original algorithm has $O(1)$-depth
  - Qubit-efficient algorithm has $\widetilde{\Theta}(n)$-depth

Original Alg.



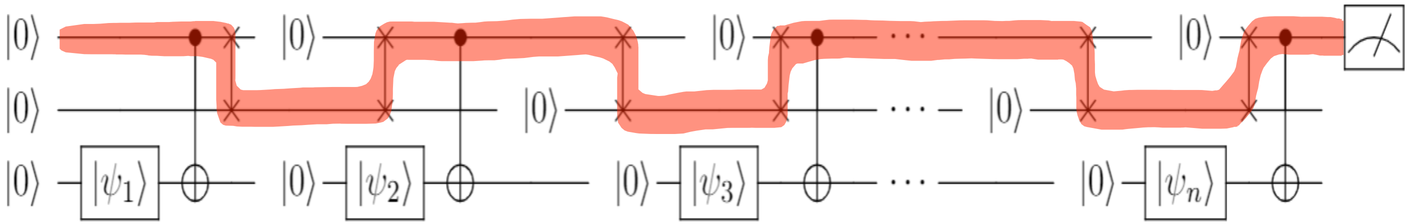
Qubit-efficient Alg.



Effective Depth

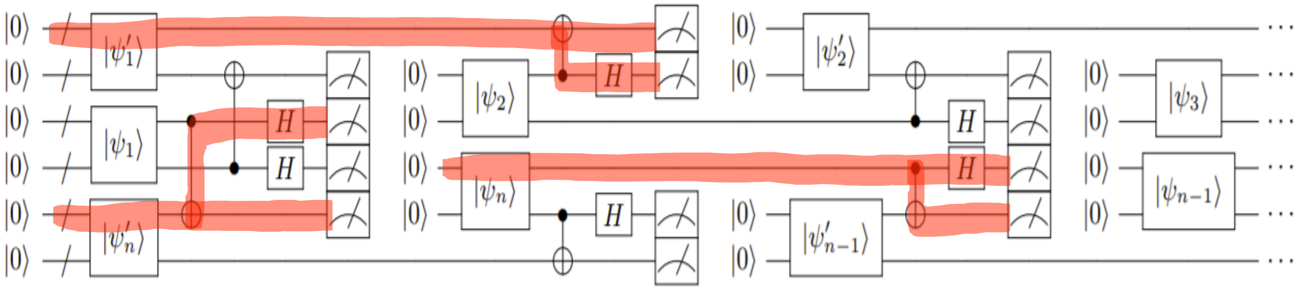
- Naïve idea: longest time any qubit goes between resets

- Counterexample:



Effective Depth

- Effective circuit depth: maximum length of a path along which there is information flow.
- Then, both the original and qubit-efficient algorithms have *effective depth* $O(1)$.



- Reduces to standard depth for circuits without resets.

Summary [arXiv:2010.03080]

- "Mid-circuit measurement and reset" is an underexplored tool that will be crucial for the utility of NISQ devices.

- Our algorithms for estimating $\mathrm{Tr}(\rho_A^n)$ require asymptotically fewer qubits but achieve similar noise resilience. This enables entanglement spectroscopy of larger quantum systems on NISQ devices than previously possible.

- *Effective circuit depth* generalizes standard depth to circuits using qubit resets. Useful for predicting noise-resilience of such circuits.

- Open Question: What other algorithms and applications can be made NISQ-ready using qubit resets?